OCR
RECOGNISING ACHIEVEMENT

Monday 13 May 2013 – Afternoon

AS GCE MATHEMATICS (MEI)

4755/01 Further Concepts for Advanced Mathematics (FP1)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4755/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 Find the values of A, B, C and D in the identity $2x(x^2 - 5) \equiv (x - 2)(Ax^2 + Bx + C) + D$. **[5]**

2 You are given that $z = \frac{3}{2}$ is a root of the cubic equation $2z^3 + 9z^2 + 2z - 30 = 0$. Find the other two roots. **[6]**

3 You are given that $\mathbf{N} = \begin{pmatrix} -9 & -2 & -4 \\ 3 & 2 & 2 \\ 5 & 1 & 2 \end{pmatrix}$ and $\mathbf{N}^{-1} = \begin{pmatrix} 1 & 0 & 2 \\ 2 & 1 & 3 \\ -\frac{7}{2} & p & -6 \end{pmatrix}$.

(i) Find the value of p. **[2]**

(ii) Solve the equation $\mathbf{N}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -39 \\ 5 \\ 22 \end{pmatrix}$. **[4]**

4 The complex number z_1 is $3 - 2\text{j}$ and the complex number z_2 has modulus 5 and argument $\frac{\pi}{4}$.

(i) Express z_2 in the form $a + b\text{j}$, giving a and b in exact form. **[2]**

(ii) Represent z_1, z_2, $z_1 + z_2$ and $z_1 - z_2$ on a single Argand diagram. **[4]**

5 You are given that $\frac{4}{(4n-3)(4n+1)} \equiv \frac{1}{4n-3} - \frac{1}{4n+1}$. Use the method of differences to show that

$$\sum_{r=1}^{n} \frac{1}{(4r-3)(4r+1)} = \frac{n}{4n+1}.$$ **[6]**

6 The cubic equation $x^3 - 5x^2 + 3x - 6 = 0$ has roots α, β and γ. Find a cubic equation with roots $\frac{\alpha}{3} + 1$, $\frac{\beta}{3} + 1$ and $\frac{\gamma}{3} + 1$, simplifying your answer as far as possible. **[7]**

Section B (36 marks)

7 Fig. 7 shows an incomplete sketch of $y = \dfrac{cx^2}{(bx-1)(x+a)}$ where a, b and c are integers. The asymptotes of the curve are also shown.

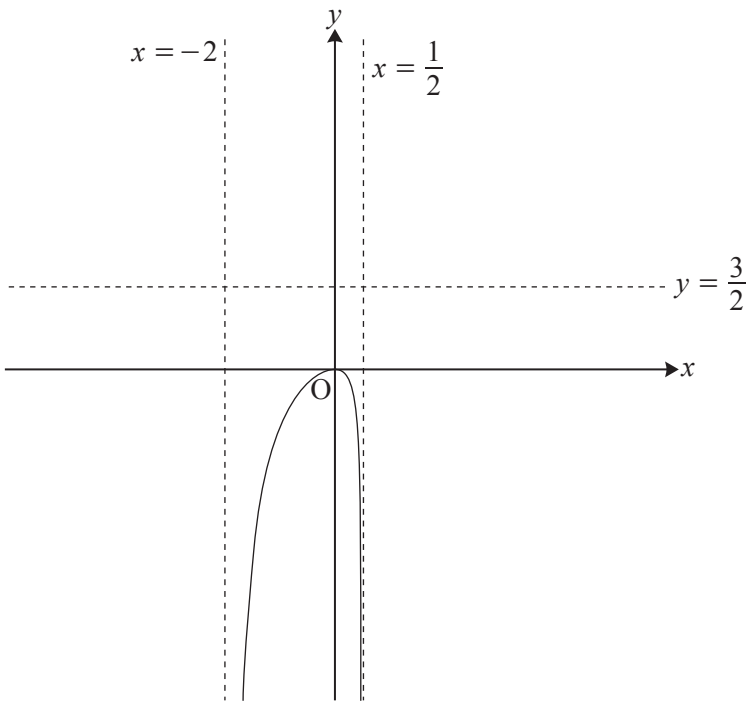

Fig. 7

(i) Determine the values of a, b and c. **[4]**

Use these values of a, b and c throughout the rest of the question.

(ii) Determine how the curve approaches the horizontal asymptote for large positive values of x, and for large negative values of x, justifying your answer. On the copy of Fig. 7, sketch the rest of the curve. **[4]**

(iii) Find the x coordinates of the points on the curve where $y = 1$. Write down the solution to the inequality $\dfrac{cx^2}{(bx-1)(x+a)} < 1$. **[4]**

8 **(i)** Use standard series formulae to show that

$$\sum_{r=1}^{n}[r(r-1)-1] = \frac{1}{3}n(n+2)(n-2). \qquad (*)$$ **[5]**

(ii) Prove (*) by mathematical induction. **[7]**

9 **(i)** Describe fully the transformation Q, represented by the matrix $\mathbf{Q}$, where $\mathbf{Q} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. **[2]**

The transformation M is represented by the matrix $\mathbf{M}$, where $\mathbf{M} = \begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix}$.

(ii) M maps all points on the line $y = 2$ onto a single point, P. Find the coordinates of P. **[2]**

(iii) M maps all points on the plane onto a single line, l. Find the equation of l. **[2]**

(iv) M maps all points on the line n onto the point $(-6,\ 6)$. Find the equation of n. **[2]**

(v) Show that $\mathbf{M}$ is singular. Relate this to the transformation it represents. **[2]**

(vi) R is the composite transformation M followed by Q. R maps all points on the plane onto the line q. Find the equation of q. **[2]**